AFRL-AFOSR-VA-TR-2018-0382

Sparse Measurements and Optimal Sensor Placement for Classification and State Estimation of Complex Systems

Jose Kutz
UNIVERSITY OF WASHINGTON

10/09/2018
Final Report

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTB1
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE			*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
08-10-2018	Final Report	09/01/2015-08/31/2018

4. TITLE AND SUBTITLE Sparse Measurements and Optimal Sensor Placement for Classification and State Estimation of Complex Systems	**5a. CONTRACT NUMBER** FA9550-15-1-0385
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) Jose Nathan Kutz	**5d. PROJECT NUMBER**
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of Washington Department of Applied Mathematics Lewis Hall 118 Seattle, WA 98195-3925	**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Force Office of Science and Research 875 Randolph Street Suite 325 Room 3112 Arlington, VA 22203	**10. SPONSOR/MONITOR'S ACRONYM(S)**
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

DISTRIBUTION A: Distribution approved for public release

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Determining the optimal placement of sensors under a cost constraint is relevant to many fields of scientific research and industry. Indeed, such considerations are critical in evaluating global monitoring systems and characterizing spatio-temporal dynamics (e.g. the brain, ocean and atmospheric dynamics, power grid networks, fluid flows, etc). For these applications, it is typical that only a limited number of measurements can be made of the system due to either prohibitive expense (i.e. either sensors are expensive, or they are expensive to place, or both) or the inability to place a sensor in a desired location (inaccessibility). Additionally, there are a number of high-level objectives for sensor placement, most of which are well studied. Common objectives include classification, reconstruction, reduced-order modeling, and control. We develop a heuristic, greedy sampling strategy whereby the sensor placement optimization is formulated as a cost-constrained problem in a relaxed form. We further introduce a parameter representing the balance between the quality of the reconstruction and the cost, and thus can evaluate a cost-error curve. The simple algorithmic structure proposed provides an effective and scalable strategy for economical sensor placement for a wide range of scientific and engineering applications.

15. SUBJECT TERMS

Sensor placement, sparse sampling, sparse optimization, data-driven sensors

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			Jose Nathan Kutz
					19b. TELEPHONE NUMBER *(Include area code)* (206) 685-3029

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18
Adobe Professional 7.0

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J. Nathan Kutz
Department of Applied Mathematics
University of Washington, Seattle, WA 98105

Final Report Prepared for
Dr. Arje Nachman, Program Manager
Physical Mathematics
875 North Randolph Road, Suite 325, Room 3112
Arlington, VA 22203

ABSTRACT

We developed new, smart sensor algorithms that are capable of maximizing information acquired from a small (sparse) number of sensors. By optimal placement, three critical sensor tasks can be enacted: (i) classification and/or categorical decision making about the data, (ii) reconstruction of the full state-space, and (iii) in the case of dynamical data, an efficient prediction of the future state. The work capitalizes on recent developments in compressive sensing and sparse representation by partnering these innovations with reduced order modeling and machine learning techniques, thus allowing for methods capable of accomplishing the above tasks with a minimal number of sensors. The work has two primary thrusts. The first is to exploit an enhanced sparsity technique to learn *spatial sensor locations* that optimally inform categorical decisions. Sensor locations may be learned from full data sets, or importantly, from a random subsample. The second objective demonstrated that sparse sampling can characterize and model complex, nonlinear dynamical systems over a range of dynamical behaviors. By constructing nonlinear libraries of the dynamics, advantage can be taken of the discrete empirical interpolation method that allows for the approximation of nonlinear terms in a sparse, low-dimensional way. The selected sampling points are shown to be nearly optimal sensing locations for characterizing the underlying dynamics, stability, and bifurcations of complex systems, thus partnering with the first objective. The method facilitates a family of local reduced-order models for each physical regime measured, thus allowing for an accurate reconstruction of the full state of the system along with a low-dimensional, and accurate, prediction of its future state, even under noisy measurements. In combination, the two primary objectives partner to offer innovative methods for optimizing a limited budget of sensors for extracting maximal information.

Contributions and innovations achieved during this proposal

In this proposal, we described a mathematical framework to exploit sparsity for classification, reconstruction and forecasting in complex systems, i.e. we optimized sensor locations for extracting meaningful information. The distinct contribution consists of optimally selecting, within a large set of measurement locations, and under cost constraints, a smaller subset of key locations that serve to perform these tasks. Our initial findings demonstrate that using a very few learned sensor locations which respect the cost constraints performs comparably with using the full state measurements. Further, our algorithms includes parameters to tune between the trade-offs between cost, fewer sensors and accuracy.

The proposal had a number of key thrusts that provided critical engineering innovations:

- **Sensor Cost:** We developed an optimization framework that allows us to explicitly account for a cost landscape for sensor placement. This requires a substantial reformulation of the optimal sensor placement problem. Within this framework, one can also evaluate how to: (i) evaluate a minimal sensor set for a given performance criteria, and (ii) explore the use of dynamic sensors that are allowed to move in the spatio-temporal system along optimal trajectories, again reducing the number of required sensors.
- **Reduced Order Models (ROMs):** We can also leverage the emerging power of dimensionality reduction, specifically the proper orthogonal decomposition to reconstruct the full state of the spatio-temporal system with only a small number of measurements in a small rank dynamical system. The preliminary results on our *sparsity enabled ROMs* architecture shows remarkable performance characteristics and has a modular framework which allows us to expand its underlying architecture for improved performance gains as well as accounting for potential cost constraints and forecasting.
- **Model Discovery for Forecasting:** The third thrust of this proposal will be to use the limited sensors for the discovery of underlying dynamical models, both linear and nonlinear. These models will serve as the basis for improved forecasting capabilities as the Kutz group has pioneered the model discovery process from time series measurements acquired from sensors. With streaming data and the results of Aim 1 and 2, this model discovery platform can serve as a state-of-the-art global monitoring and forecasting system.

Overall, this work took an engineering perspective of probing complex systems with underlying low-dimensional structure, with the explicit goal of performing some categorical decision about the state of the system along with state reconstruction and forecasting. The optimally sparse spatial sensors framework is particularly well suited for engineering applications, as an upfront investment in the learning phase allows remarkably efficient performance for all subsequent queries. Integrated together, the work has the potential to significantly upgrade the current state-of-the-art global monitoring and forecasting systems.